Confidence Intervals (Cont.)

– Normal 2-sided for Quantile (β)

$$\left(\bar{x} + k_{2,\alpha,\beta,n} s < \beta < \bar{x} + k_{2,1-\alpha,\beta,n} s\right) = 1 - \alpha$$

– Normal 1-sided for Quantile (β)

$$\left(\bar{x} + k_{1,\alpha,\beta,n} s < \beta\right) = 1 - \alpha \qquad \left(\beta < \bar{x} + k_{1,1-\alpha,\beta,n} s\right) = 1 - \alpha$$

Normal 2-sided Confidence Intervals for Quantiles

$$k_{2,\alpha,\beta,n} = r\sqrt{\frac{n-1}{\chi^2_{1-\alpha,\nu}}} \qquad k_{2,1-\alpha,\beta,n} = r\sqrt{\frac{n-1}{\chi^2_{\alpha,\nu}}}$$

$$\beta = \int_{1/\sqrt{n}-r}^{1/\sqrt{n}+r} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}\,dx = F(1/\sqrt{n}+r) - F(1/\sqrt{n}-r)$$

Given desired β must first solve bottom equation for r (requires iteration) and then given r and a desired α must solve top equations to obtain the k_2 coefficients.

Normal 1-sided Confidence Intervals for Quantiles

$$k_{1,1-\alpha,\beta,n} = \frac{t'_{1-\alpha,n-1,z_\beta\sqrt{n}}}{\sqrt{n}}$$

$$k_{1,\alpha,\beta,n} = \frac{t'_{\alpha,n-1,z_\beta\sqrt{n}}}{\sqrt{n}}$$

Noncentral *t* Distribution

Illustration – Normal 2-sided Confidence Interval for 0.95 Quantile

Data
51
72
71
49
63
33
48
51
34
61

$$\left[53.3 + (1.4982)(13.6) < \beta < 53.3 + (3.3794)(13.6)\right] = 1 - 0.05$$

$$\left(73.6 < \beta < 99.2\right) = 0.95$$

Thus, given that the data represent a random sample from a normal population, we can state that with 95% confidence the interval 73.6 – 99.2 contains the 95th Percentile of the population (on average, 95 out of 100 such random interval realizations would contain β).